Automated global segmentation of mediolateral oblique mammographic images

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Abstract. A mediolateral oblique (MLO) mammogram image contains far more than just an image of the breast. A film clip is generally present along with imaging artefacts and background noise. Before any computer analysis of the mammogram begins, the identification and extraction of the breast area is necessary – a technique called global segmentation. This paper describes in detail a fully automated algorithm for global segmentation of MLO mammograms to create an accurate breast region without user intervention.

Keywords: mediolateral oblique; mammogram; global segmentation; image processing; image analysis

1 INTRODUCTION

Image segmentation describes the process in which an image is decomposed into smaller, significant constituent parts known as *segmented objects*. The accurate segmentation of a mammogram to identify the breast area is an important pre-processing step in mammogram analysis [1], and a two-phase segmentation is a common approach. The image is initially segmented to isolate the breast area from the background, and this area is then further segmented into regions of interest for analysis.

Global segmentation is applied to the entire image and is used to separate the breast region from the non-breast (background) region. The background generally consists of a film label to identify the image, noise and other artefacts such as scratches or unexposed areas of film. Segmentation of these two regions will occur along the *breast boundary*, also known as the *breast contour* or *skin-air interface*. This boundary contains significant information relating to the differences in deformation of two contralateral mammograms and is the source of information for relating the position of the nipple relative to the skin [2].

The simplest technique for achieving global segmentation of an image is thresholding. The intensity of each pixel in the image is compared to a predefined threshold value, and set to zero intensity (black) if it is lower, or full intensity (white) if it is greater or equal. Mammogram segmentation methods in the literature often rely on thresholding the image at a fixed [3] or dynamically calculated intensity value. Many methods for finding an appropriate threshold value have been proposed. In the frequency domain, a threshold value can be extrapolated from the first peak in a smoothed histogram of the image [4] or the first valley after the first peak in the intensity histogram of the original image [5] or a median filtered image [6]. Others [7-9] simply refer to a peak detection algorithm without qualification. Wirth and Stapinski [10] used a dual threshold technique to obtain an estimate of the breast contour, establishing an initial threshold intensity value using the unimodal thresholding technique from Rosin [11]. Alternatively, inspecting the spatial image using a point-dependent technique [12] can yield an intensity threshold value by comparing the difference between neighbouring pixels [13]. Saha *et al.* [14] describe a fuzzy-connectedness segmentation technique which uses an estimate of pixel level object scale by considering the first peak in the intensity histogram as the mean background intensity and observing the symmetry of the intensities surrounding the peak. In the paper, the standard deviation of background intensities is computed as the root-mean-squared distance of the intensities and is used along with the mean intensity to compute an object-feature-based affinity.

Global thresholding alone is inadequate for obtaining accurate skin boundary segmentation, however, because background noise appears at similar grey scale intensities as subcutaneous tissue. This is a recognised limitation of global thresholding, and other researchers have investigated contour tracing [2, 15, 16] as an alternative technique and Suckling *et al.* [17] introduced a neural network to assist contour tracing. Ojala *et al* [18] used a dynamically calculated binary threshold followed by morphological smoothing to produce an initial estimate of the breast area, and then experimented with three alternative methods of Fourier transform, snake and B-spline curves to enhance the rippled boundary to restore the shape of the breast skin. In a recent paper [1], Wirth *et al* proposed a segmentation technique to remove high-intensity artefacts using morphological reconstruction [19], a method founded on the concept of geodesic transformations. While this method will eliminate the bright non-breast features that are easily visible on the image – such as the film clip – it will fail in segmenting the skin-air boundary because background noise and scratches are generally low intensity, particularly along the skin boundary.

Méndez *et al.* [9] detected the skin boundary on craniocaudal mammograms using an algorithm that calculates the grey level gradient of a smoothed image. A threshold algorithm is first applied to the original image to eliminate *unexposed and exposed non-breast regions* by selecting lower and upper threshold values from the two peaks at each end of the histogram scale. The image is then smoothed using a square 11 × 11 kernel mean-pixel value convolution and divided into three regions. Each region determines the direction of a tracking process that uses the gradient of nine preceding pixels to detect the border pixels. A pixel at co-ordinate (x, y) is taken to belong to the border if the grey-scale value, $f(x,y)$, if the nine previous pixels satisfies the condition:

$$f(x_{-9}, y_{-9}) < f(x_{-8}, y_{-8}) < \dots < f(x_{-3}, y_{-3}) \leq f(x_{-2}, y_{-2}) \leq f(x_{-1}, y_{-1}) \leq f(x,y) \qquad (1)$$

2 METHOD

The algorithm described in this paper seeks to provide an automated facility for global segmentation of mediolateral oblique mammogram images. The algorithm does have a set of configurable parameters which can be used to optimise the processing for a specific set of images, to account for variation in image acquisition environments, but once these parameters are set, the images can be automatically segmented without further user intervention.

2.1 Orient the image to nipple-right

The segmentation algorithm is a complex one, so to keep it manageable it expects that the image is oriented vertically with the nipple on the right hand side and the chest wall on the left. Mammograms oriented with the nipple on the left hand side must undergo a simple *mirror* operation to produce a vertically reflected image. After the segmentation is complete, all output images (the segmented image, the segmentation mask and the contour outline) can be reflected to re-establish a correctly oriented image to match the original.

2.2 Create an initial estimated breast region

2.2.1 Binary threshold

The first task in the procedure, to create a binary image to represent a *good proportion* of the breast region, is itself non-trivial. The definition of a *good proportion* in relation to this study must be one that incorporates all but the narrow subcutaneous tissue area. Papadopoulos *et al.* [3] claim that most pixels in a 256 level grey-scale image with intensity less than 20 will "*belong to the background area, although a small number exists belonging to the tissue area close to the breast surface*". However, in this study, using the MIAS MiniMammography database [20], a fixed threshold at 20 proved too high for a general case.

An alternative to using a fixed threshold value is to select a value for each image based upon observations made about the intensities contained within the image itself. This is known as a *dynamic* *threshold*, and many researchers have proposed varying techniques for calculating this for mammogram images. Behrenbruch *et al*. [5], for example, find the first valley in the image intensity histogram to select a threshold value and observe that this will produce an "*approximation of the* [breast] *boundary … while preserving anatomical features of the original image*". Alternatively, Richard *et al*. [Richard, 2003 #190] proposed to use the first peak in the smoothed histogram of the image to select their threshold value.

Figure 1 - Scratched mammogram image where the scratch artefact is connecting the breast region to a non-breast region.

Following extensive trials of these methods, we have concluded that no one method can successfully be used across all images in a single dataset, and it therefore follows that alternative datasets will themselves introduce further variance to invalidate the selection of a single method. A dynamic selection of two algorithms, with a per-dataset influential parameter was also investigated. The two algorithms apply an edge-crossing procedure to an intensity histogram distribution. The first applies the edge-crossing to the raw image intensity histogram and selects the value at the third crossing, and the second algorithm smoothes the histogram distribution using a three-value averaging operator before selecting the value at the third crossing. The value that is chosen to be used in the binary threshold is the minimum of the two dynamically selected values which are then adjusted to fit within a pre-configured range. The range can be used to tune the algorithm for a specific dataset; for example, the threshold value can be influenced with knowledge of the image acquisition parameters or through manual visual inspection of the dataset. Chandrasekhar *et al.* reported that from their analysis of the MIAS database, intensity levels of 10 or below belong to the background, and used an initial threshold of 12 for their work [4]. In this study, images mdb027, mdb031, mdb201, mdb246 and mdb303 produced dynamic thresholds of 8, 8, 6, 8 and 5 respectively and produced a segmentation result that was considered to be lower quality than others in the test set. Re-processing the images with a threshold of 12 produced a good result in each case. The restrictive parameters for the MIAS MiniDatabase have therefore been set to 12 for both lower and upper limit in this research. This range effectively forces a fixed threshold of 12 to be applied to images in this particular dataset; however we cannot conclude that a fixed threshold is adequate for all datasets, so the range parameters provide configuration and flexibility to accommodate the variations in mammogram images.

2.2.2 Morphological filtering

The binary threshold is likely to result in an image that contains many disjoint pixels along the breast contour where there were peaks of intensity in the subcutaneous tissue. A *morphological closing* operator [21] can be used to connect fragmented pixels to the larger breast area. A morphological closing operator is a combination of the elementary morphological operations *Erosion* and *Dilation*.

$$M \text{ closing } I = M \text{ erode } (M \text{ dilate } I) \qquad (2)$$

where I is an image and M is a structuring element of any shape and size, applied to I. A pixel in image I with coordinates at (x,y) is denoted by $I(x,y)$. In a binary image, morphological erosion will set white pixels to black, removing disjoint pixels, and dilation will set black pixels to white, combining disjoint regions together. Erosion and Dilation are defined [12, 22]

$$M \text{ erode } I(x, y) = \min \{ I(x-m_1, y-m_2) \mid \forall(m_1, m_2) \in M \} \qquad (3)$$

and

$$M \text{ dilate } I(x, y) = \max \{ I(x+m_1, y+m_2) \mid \forall(m_1, m_2) \in M \} \qquad (4)$$

Immediately following the morphological close operation, a morphological open filter is applied to remove small isolated regions and thin artefacts such as scratches which may otherwise connect breast and non-breast regions. An example of a scratch such as this can be seen in test image *mdb274* as shown in Figure 1. Morphological opening is defined

$$M \text{ opening } I = M \text{ dilate } (M \text{ erode } I) \qquad (5)$$

The morphological filtering in this research uses a square 3×3 structuring element in the morphological operations.

2.2.3 Extract the largest region

The image now contains an initial estimate of the breast region, and any number of disjoint groups of non-black pixels. The breast is expected to be the largest object in a mediolateral oblique mammogram.

A pixel p at coordinate (x, y) has four orthogonal neighbours, the 4-*neighbour* set $N_4(p)$

$$N_4(p) = \{ (x\text{-}1, y), (x, y\text{-}1), (x\text{+}1, y), (x, y\text{+}1) \} \quad (6)$$

and four diagonal neighbours,

$$N_D(p) = \{ (x\text{-}1, y\text{-}1), (x\text{+}1, y\text{-}1), (x\text{+}1, y\text{+}1), (x\text{-}1, y\text{+}1) \} \quad (7)$$

These two sets combine to give the 8-*neighbour* set $N_8(p)$

$$N_8(p) = N_4(p) \cup N_D(p) \quad (8)$$

In all of the neighbourhood definitions in equations 6–8, some pixels may fall outside of the image.

A *region* in an image can be defined as a continuously connected mass of non-black pixels, where two non-black pixels are connected if they lie within the N_8 neighbourhood of each other. Let V be the set of grey scale values used to define the region. At this point in the algorithm, the image is a binary image, thus $V = \{1\}$. Two pixels p and q with values from V are connected if q is in the set $N_8(p)$.

Using a region labelling algorithm [18], each region is identified and marked uniquely. The largest of these regions is then taken to be the breast area, and all pixels that lie outside of this region are set to black.

Region labelling algorithm

Given a binary image I of dimensions $n \times m$, initialise an associated $n \times m$ region label array r:

$$r(x, y) = \{ 0 \mid \forall(x, y) \in I \} \quad (9)$$

Initialise a region counting variable $k = 1$. Scanning the image from top left to bottom right, for each row and column perform the following:

If $I(x, y) = 0$, then do nothing

If $I(x, y) = 1$ and $I(x\text{-}1, y) = I(x, y\text{-}1) = 0$, then set $r(x, y) = k$ and $k = k + 1$. In this case, the left and upper neighbours of $I(x, y)$ do not belong to objects.

If $I(x, y) = 1$, $I(x\text{-}1, y) = 0$ and $I(x, y\text{-}1) = 1$, then set $r(x, y) = r(x, y\text{-}1)$. In this case, the upper neighbour belongs to the same object as $I(x, y)$.

If $I(x, y) = 1$, $I(x\text{-}1, y) = 1$ and $I(x, y\text{-}1) = 0$, then set $r(x, y) = r(x\text{-}1, y)$. In this case, the left neighbour of $I(x, y)$ belongs to the same object as $I(x, y)$.

If $I(x, y) = I(x\text{-}1, y) = I(x, y\text{-}1) = 1$, then set $r(x, y) = r(x\text{-}1, y)$. If $r(x\text{-}1, y) \neq r(x, y\text{-}1)$, then record the labels as equivalent. In this case both the left and upper regions belong to the same object as $I(x, y)$ although they have been labelled differently.

2.2.4 Crop top and bottom

Image corruption can introduce scan lines of non-black pixels across the width of the image region of interest. This can be seen, for example, in mammogram *mdb002*. To overcome this imaging corruption, the largest region is blurred using a 9×9 convolution mask and a binary threshold of zero is applied to set all resulting grey pixels to white. This image is then cropped both at the top and at the bottom, removing all scan lines where there are no black pixels between the left and right vertical boundaries of the region.

2.3 Trace Contour and re-sample

In many mammograms from the test set of images, the outlines created by the algorithm so far contain some sharp turns in the contour outline. However, the region growing algorithm that follows requires a smooth curve to yield a good result. Ojala *et al* noted the same observation in [23], "*the morphological filtering … produces a twisting boundary line*", and evaluated three alternatives for smoothing the line; *Fourier descriptors*, *snakes* (an energy-minimizing curve influenced by image forces), and *spline* functions.

In this research, the breast area estimate is smoothed by tracing around the outside of the region to create a single pixel contour which is then re-sampled at every 12^{th} pixel from the top most row to the left most column, producing a set of points which are subsequently connected with short straight lines. The two boundary points, p and q, are then joined via a point j such that lines pj and qj are mutually perpendicular and orthogonal to the major axis of the image. An enclosed hollow outline region is therefore created, and can be filled to visually represent the estimated breast region.

2.4 Extend the breast region to the skin-air boundary

A mammogram image shows poor definition of the breast border as background noise can generate grey scale pixels of similar intensity to those contained in the subcutaneous tissue. Further, the non-linear deformation of the breast during the acquisition of a mediolateral oblique mammogram yields a weaker contrast along the breast boundary at the axilla. In this area, the compression is reduced due to the closer proximity to the rigid chest wall, the contour on the mammogram image will fade and the boundary of the breast may not be visible at all.

The process described in section 2.2 produces an image, I_R, containing an initial estimation of the breast region that will contain all but the relatively narrow subcutaneous tissue. This initial region, which lies entirely inside the actual breast, is extended outwards towards the skin-air boundary. For each row, w, in I_R containing pixels within the estimated region, the column position of the last white pixel is recorded for row w and its adjacent rows, w-1 and w+1

$$p_1 = (x_1, w-1),\ p_2 = (x_2, w),\ p_3 = (x_3, w+1) \qquad (10)$$

From the points p_1, p_2, and p_3, the tangent to the region at row w is calculated. A vector $\mathbf{v}$ is then constructed perpendicular to the tangent to determine the direction in which the region will be extended from p_2. Assuming the origin of the image to be in the top left corner, for non-boundary conditions the vector $\mathbf{v}$ perpendicular to the tangent is defined

$$\mathbf{v} = ((w + 1 - w) - (w - 1 - w))\mathbf{i} + ((x_1 - x_2) - (x_3 - x_2))\mathbf{j}$$

$$\therefore \quad \mathbf{v} = 2\mathbf{i} + (x_1 - x_3)\mathbf{j} \qquad (11)$$

At the top and bottom boundaries, $(w - 1)$ and $(w + 1)$ respectively become w, and in both situations, equation (11) reduces to

$$\mathbf{v} = \mathbf{i} + (x_1 - x_3)\mathbf{j} \qquad (12)$$

The *extension* is a measure of the number of pixels drawn perpendicular to the tangent of the estimated breast region at each row w, from point p_2 along vector $\mathbf{v}$ towards the breast border.
The extension is measured on an image I_H that is derived from the original mammogram image thus. A morphological close operator is applied to the original image before the largest region is extracted using the algorithm in section 2.2.3. A zero-anchored histogram-equalisation is then performed, producing I_H. From a point with coordinates corresponding to the point p_2 in I_R, pixels along vector $\mathbf{v}$ are inspected. The intensity values of the pixels in the subcutaneous area of a mammogram generally decrease approaching the skin-air border [18]. The breast border is therefore identified at the point where intensity zero is found, or where the intensity is greater than that of its predecessor.

The algorithm is based on a scan line approach; each row in I_H is extended from a coordinate corresponding to point p_2 in I_R which represents the right most white pixel in the scan line. This will correctly extend the region to encompass subcutaneous tissue that is present to the right of the point; however, if subcutaneous tissue is present directly below, or below left of p_2, this will be missed. The algorithm is therefore applied both horizontally and vertically. The vertical processing from top to bottom will ensure that subcutaneous tissue below the estimated area is correctly incorporated into the final breast region.

The calculation of the extension from each row and column is crucial for the accuracy of the final contour. As the contrast is very weak in many cases, the algorithm is passed over the image twice. First, the extension from each coordinate p_2 is calculated and added to a running total. At the end of the first pass, the total is divided by the number of non-zero extensions to yield the mean average extension length, e_m. During the second pass, the extended coordinate p_e from each row and column is calculated to be distance e_m from p_2 along vector $\mathbf{v}$. The points extended horizontally and vertically from p_2 are stored separately as p_h and point p_v.

$$P_h = \{ p_h \mid \forall\, p_2 \in I_H \},\ P_v = \{ p_v \mid \forall\, p_2 \in I_V \} \qquad (13)$$

2.5 Create the breast region mask and outline

From the two sets in (13), a breast region mask can be constructed. The mask is a binary image where black pixels represent the background and non-black pixels represent the breast region. The mask is created by a simple scan line filling algorithm using the coordinates in the sets P_h and P_v.

A horizontal scan line filling algorithm is performed using the coordinates in P_h. A left margin is identified by calculating the middle non-black column in the bottom scan line in image I_R. For each coordinate in P_h, pixels in the scan line corresponding to the y component are set to non-black from the left margin column to the x component. This process is repeated vertically using coordinates in P_h and a top margin calculated from the middle non-black row in the right-most column in image I_R. Finally this image is combined with image I_R using a Boolean OR operator such that the resultant image contains non-black pixels from both images. This operator restores the rectangular region in the top left corner of the new mask, and ensures that there is no exclusion of subcutaneous tissue that was previously identified by the initial binary threshold.

Remove Protrusions and fill scan lines

To smooth the edge of the breast without cutting into subcutaneous tissue, the breast region is further manipulated in scan lines. First, protrusions are removed by scanning the image horizontally and vertically and setting to black any set of white pixels of up 7 pixels in width. Finally, any *holes* in horizontal and vertical scan lines are filled. A *hole* in this context is defined by a set of n black pixels along the scan line that contains at least one white pixel either side of the set. For each hole found in each scan line, all pixels within the hole are set to white, producing image I_M.

Breast region outline

The final breast region outline can be derived directly from the binary image I_M by applying an edge smoothing algorithm on the breast contour. A re-sampling is performed along the breast curve – from the top-most row to the left-most column. Every 25th pixel co-ordinate is extracted along this border, and a *b-spline* curve is then fitted through the sampled points to create a smooth curve between the two boundary points. Finally, the two boundary points are joined using the same technique as described in section 2.3. An enclosed hollow outline region is therefore created, and can be filled to visually represent the estimated breast region and used as a solid binary mask for further image processing.

3 EXPERIMENTAL RESULTS

The algorithm has been applied to the 322 images in the MIAS MiniMammographic database [9] and a random sample of 82 images have been analysed for reporting these initial results. In addition, a subset of images have been rotated +/–5° about the centre and then cropped to ensure a vertical edge on the chest side of the image. This test was designed to validate the algorithm's invariance to imaging parameters such as breast compression and rotation, and differences in background noise. The simple operation of image rotation will introduce new intensity levels, and in subcutaneous tissue areas where the pixel intensities are low contrast, the process can introduce a distribution of background noise that one might expect in the imaging process.

The accuracy of the algorithm was determined by visual inspection of the segmented border overlaid on I_H. For all mammograms in the test data set, I_H shows the breast outline visually distinct from the border region. For evaluation of the algorithm, the generated outline was measured against this visual outline to assess the success of the algorithm in finding a true breast-air border.

The region growing algorithm was shown to perform well in 87.8% of cases (72 images) to approximate the breast skin contour. In seven images – 8.54% of the sample – the contour was seen to be slightly outside of the visible boundary – up to 12 pixels in some cases. In the test data set, the images were digitised at 50 micron pixel edge and have been reduced to 200 micron pixel edge, so a 12 pixel margin of error represents 2.4 mm. The remaining three images in the sample – 3.66% – were unsuccessful in producing an accurate shape mask. In some images, for example mdb261 and mdb262, this error margin is large enough to encompass the very low intensity nipple region, which would otherwise have been lost.

Figure 2 – Mammogram with extensive noise and imaging artefacts (*mdb274*)

Poor image quality

Image *mdb274* is a mammogram with the breast oriented with the nipple to the right. The mammogram shows extensive noise and imaging artefacts at the right and bottom of the image (see Figure 2). High intensity noise such as this is not detrimental to the algorithm accuracy and performance, however, due to the intensity histogram distribution analysis that is performed to create the initial breast estimate (section 2.2.1). The film clip is shown as a low intensity artefact within the breast area, with high contrast numeric figures. This region is encompassed within the initial binary threshold and is therefore included in the final segmented breast area.

4 CONCLUSION

The method of mammogram segmentation presented in this paper is expensive in processing, but initial tests have shown that the algorithm produces a good breast contour that has never been seen to exclude any fibro glandular tissue area in the sample image data set. Where deviation from the true border has occurred, the identified boundary has always been outside of the true area, and therefore never excludes subcutaneous tissue.

Precise segmentation of the low intensity border is a recognised challenge, and no automated algorithm has yet been published to accomplish this. The breast outlines created herein are not perfect in all images; in particular, the algorithm will not trace the contour of the nipple in profile, nor will it create an accurate contour if there is an uneven thickness of subcutaneous tissue through the image. The selection of the global threshold for creating the initial breast estimate in section 2.2.1 is essential to the overall accuracy and success of the algorithm.

The algorithm performs well on images of good quality (defined in [24]), but lesser quality images suffer in the segmentation. There is a direct correlation between the quality of the image and the accuracy of the algorithm, specifically in the selection of the initial threshold value. It is the intent of the research to automatically perform an accurate global segmentation on good quality images. The algorithm has a number of parameters that can be adjusted to compensate for poor quality images, but this is not a part of the automated process.

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